

CHAPTER 4

COMPUTATIONAL SCHEMES

CHAPTER 4 TOPICS

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4.0 Chapter 4 Reference List

When writing, locate a description of interest in the first column; then find its matching symbol in the second column.

When reading, locate a symbol of interest in the second column; then find its matching description in the first column.

Description	Symbol
-----	-----
begin-cancellation indicator	⠄⠄⠄⠄⠄⠄ (12346)
end-cancellation indicator	⠄⠄⠄⠄⠄⠄ (246)
first component of ruled line	⠄⠄⠄⠄⠄⠄ (5)
general component of ruled line	⠄⠄⠄⠄⠄⠄ (25)
one-character cancellation indicator	⠄⠄⠄⠄⠄⠄ (456)
single-cell representation of a two-digit number	
10	⠄⠄⠄⠄⠄⠄ (1356)
11	⠄⠄⠄⠄⠄⠄ (12)
12	⠄⠄⠄⠄⠄⠄ (123)
13	⠄⠄⠄⠄⠄⠄ (125)
14	⠄⠄⠄⠄⠄⠄ (1256)
15	⠄⠄⠄⠄⠄⠄ (126)
16	⠄⠄⠄⠄⠄⠄ (1235)
17	⠄⠄⠄⠄⠄⠄ (12356)
18	⠄⠄⠄⠄⠄⠄ (1236)

4.1 Recognizing a Computational Scheme

A computational scheme can be recognized by its two-dimensional disposition on the printed page and the vertical alignment of its elements into columns clearly intended for actual or potential computation. A fraction, although its terms may be vertically aligned, is not a computational scheme because the alignment is not for the purpose of computation. Thus, a computational scheme is displayed text which must be treated as spatial material. All the examples in this chapter are computational schemes.

4.2 Format Requirements

There must always be a blank line above and a blank line below every computational scheme. The transition to a new braille page before the scheme begins or after it ends does not nullify this requirement. However, the line of hyphens that marks the transition to a new print page does not require such blank lines.

The entire computational scheme must be confined to a single braille page by employing, if necessary, every space-capturing device permitted by NUBS.

Two or more computational schemes may be brailled across the page provided that there is a blank column, at least two cells wide, between adjacent computational schemes. In determining the placement of a computational scheme, the simulated ruled lines which it contains must be taken into account to abide by this two-cell separation requirement.

The label, if any, of a computational scheme may be a number, a letter, or some other symbol or combination of symbols. Such symbols may be enclosed, punctuated, or modified in some other way. In a computational scheme, the numeric indicator should be omitted when it otherwise would be required. The label of a computational scheme is not part of the scheme. Accordingly, if the label requires the numeric indicator according to some other rule, that indicator must be supplied.

If possible, a label must be placed to the left of the computational scheme alongside the first line of original data. A carried-number line in an addition scheme, replacements of cancelled numbers in a subtraction scheme, and the quotient in a division scheme are not original data. If the label can be so placed, there must be a blank column exactly one cell wide between the label and the scheme, and the simulated ruled lines in the scheme must be taken into account in determining the placement of the scheme relative to its label.

If the label cannot be placed alongside the scheme, it must be placed above the scheme, and separated from it by exactly one blank line.

Two or more labeled computational schemes may be brailled across the page provided the two-cell separation requirement is observed between adjacent schemes.

It is not necessary for a computational scheme to display the entire computation, including the result. In many textbook exercises, the entire scheme is displayed except for the result of the computation. In many illustrative examples, a computational scheme is built up step by step so that only partial computational schemes of increasing completeness are displayed at each step.

In a computational scheme for addition or subtraction, the sign of operation (plus or minus) should not be an obstruction in any column in which computation, actual or potential, is to be performed. The sign of operation should be moved to the left as far as is necessary to avoid becoming such an obstruction. In a multiplication scheme, it is not necessary to move the times sign out of the way, since the algorithm for multiplication does not involve moving up and down columns in which the multiplication sign might be present.

Since it is inconvenient and impractical to draw a ruled line in braille to correspond to a ruled line in print, a series of dots (25) serves to simulate a horizontal ruled line in braille, except for its first component which is always dot 5. We will use the expression "ruled line" rather than the longer expression "simulated ruled line" for convenience. In the 1972 Nemeth Code, a ruled line was called a "separation line."

In an addition or subtraction scheme, the length of the ruled line is determined globally. Its first component (dot 5) must fall in the column one cell to the left of the leftmost character of the entire computational scheme. Its last component (dots 25) must fall in the column one cell

to the right of the rightmost character of the entire computational scheme. The length of a ruled line in a multiplication or long division scheme is determined locally. In a multiplication scheme, there usually is a ruled line between the multiplier and the first partial product. Only these two components of the multiplication scheme determine the length of that ruled line. There may be partial products below the first one which extend beyond the shelter of that ruled line, but this plays no role in determining its length. Furthermore, there usually is a ruled line below the last partial product. If there is no final result, then the last partial product alone determines the length of the ruled line. If there is a final result, then this result and the last partial product together determine its length. The same considerations are valid in a long division scheme where there is usually a ruled line between the dividend and the quotient and where there are additional ruled lines between each partial product and the partial remainder that results from subtracting this partial product from the previous remainder. The purpose of the extra length of a ruled line at both ends is to distinguish it from a series of 3's.

All of the statements above are valid both for arithmetic and algebraic computational schemes. In an arithmetic multiplication scheme, the partial products form a "stairway" arrangement in which one moves to the left as he descends the "stairs." In an arithmetic long division scheme as well as in an algebraic multiplication or division scheme, the "stairway" is oppositely oriented so that one proceeds to the right as he descends the "stairs."

EXAMPLE (showing compliance with format requirements)

$$\begin{array}{r}
 5. \quad 963 \\
 + \quad 72 \\
 \hline
 \end{array}
 \quad
 \begin{array}{cccc}
 \cdot\cdot & \cdot\cdot & \cdot\cdot & \\
 & \cdot\cdot & \cdot\cdot & \cdot\cdot \\
 & & \cdot\cdot & \cdot\cdot & \cdot\cdot \\
 & & & \cdot\cdot & \cdot\cdot & \cdot\cdot \\
 & & & & \cdot\cdot & \cdot\cdot & \cdot\cdot
 \end{array}$$

This example is a simple addition scheme with a label. It is delimited by blank lines above and below it. Its label has been placed to the left of the first line of original data of the scheme and a blank column exactly one cell wide separates the scheme from its label. Thus, the last character of the label is in cell 3 while the first component of the ruled line is in cell 5. The result of the addition is not shown. Thus, this is a partial computational scheme. If the result were shown, it would be four digits long. This is longer than any of the terms in the addition scheme. It is not the transcriber's mission to anticipate the amount of space the result would occupy so as to adjust the length of the ruled line accordingly. The ruled line has the correct length relative to what is explicitly shown in this computational scheme. Numeric indicators have been omitted. The plus sign has been moved to the left so as not to obstruct any of the columns that participate in the addition. The ruled line has the correct length.

4.3 Help for Young Children

4.3.1 Column Separation

Young children, because their fine motor skills are still developing, sometimes wander from one column of figures to an adjacent one and back in the course of working with a computational scheme. For this reason, a teacher may find it expedient to separate adjacent columns of figures by a blank column which is one or two cells wide to minimize this "wandering" behavior. Ruled lines, however, must be completely formed. This "trick" has the additional advantage of allowing the insertion of a comma or a decimal point at appropriate

places as may be required. In such cases, this method of separating columns of figures is suitable for adults as well as for children.

EXAMPLE (showing column separation)

$$\begin{array}{r}
 2.54 \\
 \times 16.7 \\
 \hline
 1778 \\
 1524 \\
 254 \\
 \hline
 42.418
 \end{array}$$

This is a fifth-grade multiplication example. Since there are two digits to the right of the decimal point in the multiplicand and one digit to the right of the decimal point in the multiplier, the decimal point has been placed in the product so that it is followed by three digits. This technique is not confined to multiplication schemes; it may be used in any computational scheme when appropriate.

4.3.2 Avoiding Paper Creep

People who use a braille in the computation of a long division example (strongly recommended) frequently experience the "creeping paper" problem. As the paper is rolled up to the quotient line to enter the latest digit or the latest algebraic term and then rolled back down to the current work area, there is a tendency for the paper to "creep" so that the new braille entry is too close to the previous braille entry for comfortable reading. With a repetition of this "roll up and roll down" activity as each new digit or term is developed, which is also time-consuming, this "creeping" behavior finally causes the new braille and the old braille to merge so that both are illegible. To solve this problem, it is suggested that each new digit or term, as it is developed, be entered against the left margin abreast of the partial product about to be generated. Thus, the paper is always advanced and never rolled back. When the computation is complete, the digits or terms of the quotient can be read from top to bottom of the paper against its left margin. The final remainder is also available on the last line of the computation. If desired, these can now be transferred to the quotient line. The student should be informed that this is a "braille-only" ploy to avoid "paper creep." If the work is to be handed to a sighted teacher who knows braille, or if it is to be turned into writing by a vision teacher for submission to a subject-matter teacher, the quotient line must be supplied. Notice the NUBS method for showing the format of a long-division problem.

EXAMPLES (avoiding "paper creep")

$$\begin{array}{r}
 \underline{121 \text{ R}34} \\
 39 \text{) } 4753 \\
 1 \quad 39 \\
 \quad 85 \\
 2 \quad 78 \\
 \quad \quad 73 \\
 1 \quad \quad 39 \\
 \text{R} \quad \quad 34
 \end{array}$$

$$\begin{array}{r}
 \underline{4x^2-6xy+9y^2} \\
 2x+3y \text{) } 8x^3 \qquad \qquad +27y^3 \\
 4x^2 \quad 8x^3+12x^2y \\
 \quad \quad -12x^2y \qquad +27y^3 \\
 -6xy \quad -12x^2y-18xy^2 \\
 \quad \quad \quad +18xy^2+27y^3 \\
 +9y^2 \quad \quad +18xy^2+27y^3 \\
 \qquad \qquad \qquad \qquad \qquad \qquad 0
 \end{array}$$

These two examples are fictitious homework papers of fictitious blind children. One is a third or fourth grader studying arithmetic; the other is an eighth or ninth grader studying elementary algebra. They both place their work against the left margin since there is no reason to indent. They omit the ruled lines that are usually present in a worked-out textbook presentation except for the ruled line above the dividend. As each new digit is determined in the arithmetic example or as each new term is determined in the algebra example, it is placed against the left margin instead of on the quotient line as sighted children are taught to do. They have been told that this is a "braille trick" to avoid rolling the paper up and down. Upon completing the computation, the digits or terms are read from top to bottom and copied into the quotient line.

4.4 Carried-Number Line in Addition

When imparting to young children the skill of adding columns of numbers, an intermediate phase in the development of that skill is to have the children write down the number that must be carried from one column to the neighboring column on the left, whenever such a carry is necessary. When the addition skill is well established, these numbers, of course, are processed mentally rather than written down.

In print, carried numbers are usually shown as reduced in size and appear directly above the top entry of the column to which they apply. In this System, we devise a "carried-number line." Each component of the "carried-number line," except the first, consists of dots 2356. The first component of the carried-number line consists of dots 56. Its length is the same as the length of the ruled line at the bottom of the addition scheme.

Children must be made aware of the fact that the "carried-number line" is a "braille-only device." The print method of showing carried numbers should be explained to them.

EXAMPLE (showing a carried-number line)

$$\begin{array}{r}
 11 \\
 368 \\
 \hline
 572 \\
 940
 \end{array}$$

Since the carried-number line is longer at both ends than the terms that participate in the addition example, and since its first component is different from the remaining components, the carried-number line cannot be confused with a string of 7's as a part of the example.

4.5 Cancellation When Subtracting

When imparting to young children the skill of subtracting one number from another, an intermediate phase in the development of that skill is to have the children actually write down the numbers that play a role in a procedure called "renaming" (sometimes called "borrowing"). The form of this renaming procedure involves the cancellation of some numbers and their replacement by others. When the subtraction skill is well established, the renaming process is carried out mentally. Until then, the children must pass through this intermediate stage.

In print, we can, with one stroke (no pun intended), simultaneously show the extent of the cancellation and preserve the readability of what has been cancelled. In braille, such a feat is not easily achieved.

If a young child finds it difficult to enter this two-digit number all at once using this "trick," he can enter it in two steps: first, enter the unit's digit as a dropped number; then backspace and enter dot 1. These two steps can also be reversed.

In NUBS, we do not do anything to cancel a digit. Simply brailleing its replacement number on the braille line above it implies the cancellation of the digit. Thus, its form is not changed and it remains readable.

In the process of subtraction, replacement digits may be required at more than one level above the subtraction problem. This occurs when a "renamed digit" requires renaming as the process moves leftward across the subtraction columns.

When the renaming procedure has been completed, the actual subtraction is performed. In each column, the digit in the bottom number is subtracted from the topmost number in its column. Because of the renaming that was done before the actual subtraction was undertaken, the subtraction will always be possible. In each column, the topmost entry will be either a compressed two-digit number or a single digit.

Some additional examples of this procedure are given in the next section.

4.5.1 Teacher Alert

You should show your students how cancellation is effected in print and explain to him how the braille method is a modification of the print method. You should explain to your students who use this method that the two-digit compression "trick" is a "braille-only trick" whose use is restricted to subtraction schemes in which the renaming procedure must be shown. You should explain to your students that a sighted person can easily squeeze a two-digit number into the space originally allotted for one-digit numbers, but since this cannot be done in braille, we have to resort to this "two-digit compression trick."

This new method for handling subtraction schemes should be thoroughly field tested before it becomes an official part of NUBS. Transcribers should react to the ease of its implementation; readers should react to its readability; students, especially young children, should react to its efficiency compared with the cancellation provision in the 1972 Nemeth Code.

EXAMPLES (showing a revised cancellation technique for subtraction)

1	9,548	⠠⠠	⠠⠠⠠⠠
	<u>-6,305</u>		⠠⠠⠠⠠
	3,243		⠠⠠⠠⠠

In this subtraction scheme, subtraction is immediately possible in every column. Therefore, there are no cancelled digits and no replacement numbers.

$$\begin{array}{r}
 2 \quad \quad \quad \begin{array}{cccc} & 7 & 12 & 5 & 13 \\ & 8 & 2 & 6 & 3 \\ - & 2 & 8 & 3 & 6 \\ \hline & 5 & 4 & 2 & 7 \end{array}
 \end{array}$$

In this example, only one line of numbers above the minuend is required. If we number the columns 1 to 4 from right to left, we find that the compressed two-digit number 13 is in column 1 and the two-digit compressed number 12 is in column 3. We now can complete the problem by subtracting 6 from 13, 3 from 5, 8 from 12, and 2 from 7, entering each result in the remainder line. Since, after the renaming procedure, subtraction is possible in each column, we can even proceed to perform the individual subtractions from left to right, thereby avoiding backspacing when using the braillewriter.

$$\begin{array}{r}
 3 \quad \quad \quad \begin{array}{cccc} & 4 & 5 & 6 & 7 \\ - & 2 & 9 & 6 & 8 \\ \hline & & & & \end{array}
 \end{array}$$

$$\begin{array}{r}
 3a \quad \quad \quad \begin{array}{cccc} & 5 & 17 & & \\ & 4 & 5 & 6 & 7 \\ - & 2 & 9 & 6 & 8 \\ \hline & & & & \end{array}
 \end{array}$$

$$\begin{array}{r}
 3b \quad \quad \quad \begin{array}{cccc} & & 15 & & \\ & 4 & 5 & 6 & 7 \\ - & 2 & 9 & 6 & 8 \\ \hline & & & & \end{array}
 \end{array}$$

$$\begin{array}{r}
 3c \quad \quad \quad \begin{array}{cccc} & & 14 & 15 & \\ & 3 & 4 & 5 & 17 \\ & 4 & 5 & 6 & 7 \\ - & 2 & 9 & 6 & 8 \\ \hline & 1 & 5 & 9 & 9 \end{array}
 \end{array}$$

Example 3 is a proposed subtraction scheme; the student's task is to compute the remainder after the subtraction.

Example 3a is a "snapshot" of the status of the computation after having processed the ones column. Whenever the subtraction cannot be performed immediately, a renaming process occurs. In this case, 8 cannot be immediately subtracted from 7; therefore, the 7 in the current column is cancelled and is replaced (above) by a number which is 10 larger than itself, namely, 17. Then, the 6 in the tens column is cancelled and replaced (above) by a number which is 1 less than itself, namely, 5. Note that 4 thousands plus 5 hundreds plus 5 tens plus 17 ones has the same value as 4 thousands plus 5 hundreds plus 6 tens plus 7 ones as

NUBS offers a mechanism for showing cancellation in which the extent of the cancellation is precise and the cancelled material remains readable. We provide the following cancellation indicators:

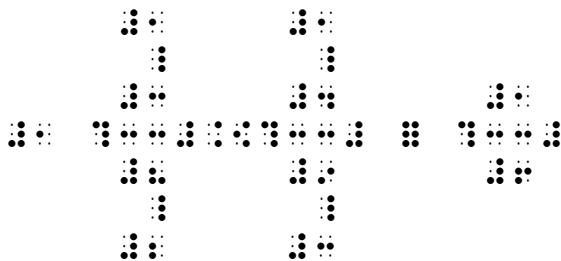
begin cancellation	⋮	(12346)
end cancellation	⋮	(246)
one-character cancellation	⋮	(456)

RULES (for cancellation)

- a *Text in which cancellation occurs must be presented in displayed form. Text containing cancellation is spatial; it must be separated from surrounding text by a blank line above and below it.*
- b *If the text to be cancelled occupies one character, the one-character cancellation indicator must be placed above or below it, as appropriate. If the text to be cancelled occupies more than one cell, the begin-cancellation indicator and the end-cancellation indicator must be placed at the first and the last cell, respectively, of the text to be cancelled; these indicators should be placed above or below the text to be cancelled, as appropriate.*

EXAMPLES (showing cancellation)

$$1 \quad \frac{\overset{1}{\cancel{3}}}{\underset{2}{8}} \times \frac{\overset{1}{\cancel{4}}}{\underset{3}{9}} = \frac{1}{6}$$



$$2 \quad (x-a_1)(x-a_2)\cancel{(x-a_3)}(x-a_4) = 0$$



Example 1 shows how two fractions are multiplied. The 3 in the first numerator cancels the 9 in the second denominator. These cancelled numbers are replaced by 1 and 3 respectively. The number 8 in the first denominator and the 4 in the second numerator have a common factor of 4. These numbers are cancelled and replaced by the result obtained by dividing each one by 4. The result is obtained by multiplying the two new numerators and the two new denominators.

Example 2 shows the product of four algebraic expressions in which the third factor has been cancelled without a replacement.

4.7 The Abacus and the Calculator

The abacus and the talking calculator have become staples in the math curriculum of blind children. When they are properly used, they yield results with the speed and accuracy that matches or even surpasses the speed and accuracy attained by sighted people. Therefore, they are valuable aids when put into the hands of blind users.

But they also pose a clear danger to the development of math skills by children if they are introduced too early. Before learning to use the abacus or the calculator, a blind child must become comfortable with the pencil-and-paper algorithms for performing the basic arithmetic operations of addition, subtraction, multiplication and division. Without this background, he will be at a loss to understand a phrase like "put down the 3 and carry a 2" when performing addition. He will have no clear conception of the process of borrowing or renaming when performing subtraction. He will have no experience with the "stairstep" pattern produced by the partial products when performing multiplication. He will have no concept of the phrase "bring down the next figure" when performing long division. All of these ideas will arise as the blind child acquires the pencil-and-paper skills that his sighted peers learn.

The ideal tool for teaching these skills is the Perkins braille. With this braille, alignment of figures is easily achieved. The two-dimensional layout of the algorithm is immediately apparent. A permanent record on paper is produced for later review.

When the pencil-and-paper skills are acquired in the performance of arithmetic operations, they are easily transferred to analogous algebraic operations. The abacus and the calculator are restricted to arithmetic operations only; they are useless in the performance of algebraic operations.

Therefore, access to the abacus and the calculator should be deferred until after the blind child has acquired the skills described above. He will then have both the skill and the knowledge of the algorithms underlying the basic arithmetic operations and the means for achieving the speed and accuracy required in everyday activities.

CHAPTER 5 EXTENSION TO MATHEMATICS

Mathematics is part of the natural sciences. These encompass the fields of mathematics, physics, chemistry, and the various branches in each of these fields. They exclude the life sciences and the humanities. There are no rigid boundaries among these fields, so that the notation in any of these fields will find its way into some other field. For example, the notational constructs in organic chemistry are frequently found in the literature that deals with microbiology. The symbols of mathematics are prominent in the field of physical chemistry.

This chapter concentrates on the extension to mathematics. We turn our attention to the field of mathematics first because its notation, to a large extent, permeates all of the other natural sciences.

In Chapter 3, we proposed a whole set of symbols that are not only mathematical, but which have found their way into the general literature. We also undertook a cursory analysis of fractions and radicals, and we addressed the basics of subscripts, superscripts and related matters. There are, however, other symbols that are found almost exclusively in the field of mathematics and its various branches. One of our guiding principles in Chapter 1, (principle h) requires that notation be represented without regard to its meaning. Therefore, if a symbol commonly found in one branch of mathematics happens to appear in another branch, or even in another field, but with a different meaning, this will have no effect on how it is represented in braille.

First, we include some symbols that were deliberately left out in earlier chapters. In those chapters we addressed the kind of text likely to be found in everyday reading material. Therefore, we presented only the basics in each covered category. Here we supply more details for each of those categories. We then go on to categories of a more technical nature which were not even mentioned in earlier chapters.

CHAPTER 5 TOPICS

5.0 Mathematics Symbol Set

- 5.0.1 Arrows (continued)
- 5.0.2 ASCII Character Set
 - 5.0.2.1 Auxiliary Notation
- 5.0.3 Calculus and Analysis
- 5.0.4 Comparison Signs (continued)
- 5.0.5 Computer and Calculator
- 5.0.6 Fractions and Radicals (continued)
- 5.0.7 Functions and Their Abbreviations
- 5.0.8 Geometry
 - 5.0.8.1 Geometric Modifiers
 - 5.0.8.2 Geometric Shapes
- 5.0.9 Grouping Signs (continued)
- 5.0.10 Indicators (continued)
- 5.0.11 Logic and Set Theory
- 5.0.12 Modifiers

- 5.0.13 Operation Signs (continued)
- 5.0.14 Punctuation (continued)
- 5.0.15 Special Letters (continued)
 - 5.0.15.1 Crossed
 - 5.0.15.2 Global Operation Signs
 - 5.0.15.3 Hebrew
 - 5.0.15.4 Miscellaneous
 - 5.0.15.5 Number Sets
 - 5.0.15.6 Obsolete Greek
 - 5.0.15.7 Variant Greek
- 5.0.16 Trigonometry
- 5.1 Arrows (continued)
- 5.2 The ASCII Character Set
 - 5.2.1 Auxiliary Notation
- 5.3 Calculus and Analysis
- 5.4 Comparison Signs (continued)
- 5.5 Computer and Calculator
- 5.6 Delimiters (continued)
- 5.7 Fractions and Radicals (continued)
 - 5.7.1 Fractions of Higher Order
 - 5.7.2 Spatial Representation of Fractions
 - 5.7.3 Continued Fractions
 - 5.7.4 Indexed Radicals
 - 5.7.5 Radicals of Higher Order
 - 5.7.6 Spatial Representation of Radicals
 - 5.7.7 Fractions and Radicals Together in a Spatial Representation
- 5.8 Functions and Their Abbreviations
 - 5.8.1 Spacing with Function Abbreviations
- 5.9 Geometry
 - 5.9.1 Geometric Modifiers
 - 5.9.2 Geometric Shapes
- 5.10 Grouping Signs (continued)
- 5.11 Logic and Set Theory
- 5.12 Special Letters (continued)
 - 5.12.1 Crossed
 - 5.12.2 Global Operation Signs
 - 5.12.3 Hebrew
 - 5.12.4 Miscellaneous
 - 5.12.5 Number Sets
 - 5.12.6 Obsolete Greek
 - 5.12.7 Variant Greek
- 5.13 Subscripts and Superscripts (continued), and Modifiers
 - 5.13.1 Primes Together with Indices
 - 5.13.2 Simultaneous and Non-Simultaneous Indices
 - 5.13.3 Left Indices
 - 5.13.4 Numeric Subscripts
 - 5.13.5 Higher-Order Indices

5.13.6 Overscripts and Underscripts

5.13.7 Modifiers

5.14 Trigonometry

5.0 Mathematics Symbol Set

This Mathematics Symbol Set contains symbols found in texts that are more technical. Symbols that are likely to be found in everyday texts are listed in the Basic Literary Symbol Set. The mathematics symbols listed below have been sorted into categories and the categories have been alphabetized.

When writing, locate a description of interest in the first column; then find its matching symbol in the second column for braille or in the last column for print.

When reading, locate a symbol of interest in the second column for braille or the last column for print; then find its matching description in the first column.

The absence of a tactile graphic in the last column, except in the case of function abbreviations, signifies an indicator.

Description	Braille	Dots	Graph

5.0.1 ARROWS (continued)			
northeast	⠠⠠⠠⠠	(4,45,135)	!
northwest	⠠⠠⠠⠠	(4,45,246)	!
northwest-southeast	⠠⠠⠠⠠	(4,56,123456)	!
southeast	⠠⠠⠠⠠	(4,56,135)	!
southwest	⠠⠠⠠⠠	(4,56,246)	!
southwest-northeast	⠠⠠⠠⠠	(4,45,123456)	!
5.0.2 ASCII CHARACTER SET (excludes letters and digits)			
ampersand	⠠⠠	(4,12346)	&
apostrophe	⠠	(3)	'
asterisk	⠠⠠	(456,246)	*
at sign	⠠⠠	(4,1)	@
backslash	⠠⠠	(456,16)	\
caret	⠠⠠	(6,26)	^
colon	⠠	(25)	:
comma	⠠	(16)	,
crosshatch	⠠⠠	(46,3456)	#
dollar sign	⠠⠠	(4,234)	\$
equals	⠠	(123456)	=
exclamation	⠠	(235)	!
grave accent	⠠⠠	(4,26)	`
hyphen	⠠	(36)	-
left angle bracket	⠠⠠	(4,126)	<
left brace	⠠⠠	(46,12356)	{
left bracket	⠠⠠	(4,12356)	[
left parenthesis	⠠	(12356)	(

percent sign	⠠⠠⠠⠠	(46,356)	%
period	⠠⠠	(12456)	.
plus	⠠⠠	(346)	+
question mark	⠠⠠	(236)	?
quote	⠠⠠⠠	(6,2356)	“
right angle bracket	⠠⠠⠠	(4,345)	>
right brace	⠠⠠⠠	(46,23456)	}
right bracket	⠠⠠⠠	(4,23456)]
right parenthesis	⠠⠠	(23456))
semicolon	⠠⠠	(23)	;
slash	⠠⠠⠠	(456,34)	/
space		(no dots)	
tilde	⠠⠠⠠	(46,35)	~
underscore	⠠⠠⠠	(6,36)	_
vertical bar	⠠⠠⠠	(4,1256)	

5.0.2.1 AUXILIARY NOTATION

begin upper-case passage	⠠⠠⠠⠠⠠	(6,6,56)	
end upper-case passage	⠠⠠⠠	(6,3)	
upper case, one letter	⠠⠠	(6)	
upper case, one word	⠠⠠⠠	(6,6)	

5.0.3 CALCULUS AND ANALYSIS

absolute value	⠠⠠⠠	(4,1256)	
del	⠠⠠⠠	(456,245)	∇
double vertical bar	⠠⠠⠠⠠	(4,4,1256)	
infinity	⠠⠠⠠	(4,236)	∞
integral	⠠⠠	(2346)	∫
large boldface pi	⠠⠠⠠⠠	(456,6,1234)	π
large boldface sigma	⠠⠠⠠⠠	(456,6,234)	Σ
limit	⠠⠠⠠⠠		lim
logarithm	⠠⠠⠠⠠		log
lower limit	⠠⠠⠠⠠⠠⠠⠠		<u>lim</u>
nabla	⠠⠠⠠	(456,245)	∇
natural logarithm	⠠⠠⠠		ln
norm	⠠⠠⠠⠠	(4,4,1256)	
partial derivative	⠠⠠⠠	(4,1246)	∂
prime	⠠⠠	(3)	'
upper limit	⠠⠠⠠⠠⠠⠠⠠		<u>lim</u>
vertical bar	⠠⠠⠠	(4,1256)	

5.0.4 COMPARISON SIGNS (continued)

approximately equal	⠠⠠⠠	(456,35)	≈
contains the element	⠠⠠⠠⠠	(4,4,24)	Σ
contains the subset	⠠⠠⠠⠠	(4,4,13456)	⊃

does not contain the element	⋮⋮⋮	(46,4,24)	∉
does not include	⋮⋮⋮	(46,4,13456)	⊄
greater than or equal	⋮⋮	(456,345)	≥
implies	⋮⋮⋮	(4,4,13456)	⊃
includes	⋮⋮⋮	(4,4,13456)	⊃
is an element	⋮⋮⋮	(4,4,15)	∈
is a subset	⋮⋮⋮	(4,4,12346)	⊆
is not an element	⋮⋮⋮	(46,4,15)	∉
is not a subset	⋮⋮⋮	(46,4,12346)	⊈
less than or equal	⋮⋮	(456,126)	≤
not equal to	⋮⋮	(46,123456)	≠
not greater than	⋮⋮	(46,345)	⋮
not less than	⋮⋮	(46,126)	⋮

5.0.5 COMPUTER AND CALCULATOR

begin keytop legend	⋮	(246)
begin screen display	⋮	(12346)
runover indicator	⋮	(456)

5.0.6 FRACTIONS AND RADICALS (continued)

complexity indicator for fractions and radicals	⋮	(6)	
component of a radical ascender	⋮	(456)	
component of spatial fraction bar	⋮	(25)	
component of a spatial vinculum	⋮	(25)	
last component of radical ascender	⋮	(56)	
radical sign	⋮	(345)	√
radical sign without vinculum	⋮⋮⋮	(46,46,345)	√

5.0.7 FUNCTIONS AND THEIR ABBREVIATIONS

amplitude	amp
antilogarithm	antilog
arc	arc
argument	arg
cologarithm	colog
cosine + isin	cis
coversine	covers
curl	curl
determinant	det
dimension	dim
divergence	div
error function	erf
exponential	exp
exsecant	exsec
gradient	grad

haversine	hav
imaginary	im
infimum	inf
limit	lim
logarithm	log
lower limit	<u>lim</u> ⋮⋮ ⋮⋮ ⋮⋮
maximum	max
minimum	min
modulo	mod
modulus	mod
natural logarithm	ln
rank	rnk
real	re
supremum	<u>sup</u>
upper limit	<u>lim</u> ⋮⋮ ⋮⋮ ⋮⋮
versine	vers

5.0.8 GEOMETRY

5.0.8.1 GEOMETRIC MODIFIERS

full line	⋮⋮	(4,123456)	←—→
half lines			
closed left	⋮⋮ ⋮	(456,4,135)	●—→
closed right	⋮⋮ ⋮	(456,4,246)	←—●
open left	⋮⋮ ⋮	(46,4,135)	○—→
open right	⋮⋮ ⋮	(46,4,246)	←—○
line segments			
left closed, right closed	⋮⋮ ⋮	(456,456,156)	●—●
left closed, right open	⋮⋮ ⋮	(456,46,156)	●—○
left open, right closed	⋮⋮ ⋮	(46,456,156)	○—●
left open, right open	⋮⋮ ⋮	(46,46,156)	○—○

5.0.8.2 GEOMETRIC SHAPES

angle	⋮⋮ ⋮	(456,5,2346)	∠
arc, concave	⋮⋮ ⋮	(456,5,1)	∪
arc, convex	⋮⋮ ⋮	(456,5,3)	∩
circle	⋮⋮ ⋮	(456,5,14)	○
diamond	⋮⋮ ⋮	(456,5,145)	
ellipse	⋮⋮ ⋮	(456,5,15)	∏
equilateral triangle	⋮⋮ ⋮	(456,5,25)	!
hexagon	⋮⋮ ⋮	(456,5,235)	
is parallel	⋮⋮ ⋮	(456,5,123)	
is perpendicular	⋮⋮ ⋮	(456,5,1234)	
octagon	⋮⋮ ⋮	(456,5,236)	

oval	⠠⠠⠠	(456,5,135)
parallelogram	⠠⠠⠠	(456,5,1245)
pentagon	⠠⠠⠠	(456,5,26)
quadrilateral	⠠⠠⠠	(456,5,12345)
rectangle	⠠⠠⠠	(456,5,1235)
rhombus	⠠⠠⠠	(456,5,125)
right angle	⠠⠠⠠	(456,5,1456)
right triangle	⠠⠠⠠	(456,5,1236)
square	⠠⠠⠠	(456,5,256)
star	⠠⠠⠠	(456,5,234)
trapezoid	⠠⠠⠠	(456,5,1356)
triangle	⠠⠠⠠	(456,5,2345)

5.0.9 GROUPING SIGNS (continued)

barred signs

braces

left	⠠⠠⠠	(456,46,12356)	{
right	⠠⠠⠠	(456,46,23456)	}

brackets

left	⠠⠠⠠	(456,4,12356)	[
right	⠠⠠⠠	(456,4,23456)]

boldface signs

brackets

left	⠠⠠⠠	(4,456,12356)	[
right	⠠⠠⠠	(4,456,23456)]

vertical bar

left and right	⠠⠠⠠	(4,456,1256)	
----------------	-----	--------------	--

double vertical bars

left and right	⠠⠠⠠	(4,4,1256)	
----------------	-----	------------	--

extended signs

braces

left	⠠⠠⠠	(46,6,12356)
right	⠠⠠⠠	(46,6,23456)

brackets

left	⠠⠠⠠	(4,6,12356)
right	⠠⠠⠠	(4,6,23456)

parentheses

right	⠠⠠	(6,12356)
left	⠠⠠	(6,23456)

transcriber's enclosures

left	⠠⠠⠠	(46,6,235)
right	⠠⠠⠠	(46,6,256)

vertical bar		
left and right	⠠⠨⠠	(4,6,1256)
half-brackets		
lower left	⠠⠨⠠⠨	(4,56,12356)
lower right	⠠⠨⠠⠨	(4,56,23456)
upper left	⠠⠨⠠⠨	(4,45,12356)
upper right	⠠⠨⠠⠨	(4,45,23456)
vertical bar		
left and right	⠠⠨⠠	(4,1256)

5.0.10 INDICATORS (continued)

begin keytop legend	⠠⠨	(246)
begin screen display	⠠⠨	(12346)
cancellation		
begin	⠠⠨	(12346)
end	⠠⠨	(246)
one character	⠠⠨	(456)
complexity indicator for		
fractions and radicals	⠠⠨	(6)
no boundary	⠠⠨	(4)
overscript	⠠⠨⠠	(45,12346)
overstrike	⠠⠨⠠	(5,12346)
runover	⠠⠨	(456)
shaded shape	⠠⠨	(46)
solid shape	⠠⠨	(456)
underscript	⠠⠨⠠	(56,12346)

5.0.11 LOGIC AND SET THEORY

Cartesian product	⠠⠨⠠	(46,246)	'
contains the element	⠠⠨⠠⠨	(4,4,24)	∈
contains the subset	⠠⠨⠠⠨	(4,4,13456)	⊃
does not contain the element	⠠⠨⠠⠨	(46,4,24)	∉
does not contain the subset	⠠⠨⠠⠨	(46,4,13456)	⊄
empty set	⠠⠨⠠	(456,356)	∅
for all	⠠⠨⠠⠨	(456,456,3)	∀
global Cartesian product	⠠⠨⠠⠨	(456,46,246)	'
global intersection	⠠⠨⠠⠨	(456,4,134)	∩
global product	⠠⠨⠠⠨	(456,6,1234)	
global sum	⠠⠨⠠⠨	(456,6,234)	
global union	⠠⠨⠠⠨	(456,4,136)	∪
if and only if	⠠⠨⠠	(4,123456)	iff
implies	⠠⠨⠠⠨	(4,4,13456)	⊃
intersection	⠠⠨⠠	(4,134)	∩
is an element of	⠠⠨⠠⠨	(4,4,15)	∈
is implied by	⠠⠨⠠⠨	(4,4,12346)	⊃
is not an element of	⠠⠨⠠⠨	(46,4,15)	∉

is not a subset of	⠠⠠⠠⠠	(46,4,12346)	$\not\subseteq$
is a proper subset of	⠠⠠⠠⠠	(456,4,12346)	\subsetneq
is a subset of	⠠⠠⠠⠠	(4,4,12346)	\subseteq
join	⠠⠠⠠	(4,1236)	\wedge
meet	⠠⠠⠠	(4,1234)	\vee
there exists	⠠⠠⠠⠠	(456,456,26)	\exists
there exists uniquely	⠠⠠⠠⠠	(456,46,26)	$\exists!$
union	⠠⠠⠠	(4,136)	\cup

5.0.12 MODIFIERS

acute accent	⠠⠠	(4,35)	'
asterisk	⠠⠠⠠	(456,246)	*
breve	⠠⠠⠠⠠	(456,5,1)	
circumflex	⠠⠠⠠	(6,26)	$\hat{}$
diaeresis	⠠⠠⠠	(46,25)	¨
dot	⠠⠠	(12456)	.
grave	⠠⠠⠠	(4,26)	`
hat	⠠⠠⠠	(6,26)	$\hat{}$
left arrow	⠠⠠⠠	(4,246)	\leftarrow
macron	⠠⠠	(156)	-
right arrow	⠠⠠⠠	(4,135)	\rightarrow
ring	⠠⠠⠠	(46,346)	°
tilde	⠠⠠⠠	(46,35)	~
two-way arrow	⠠⠠⠠	(4,123456)	\leftrightarrow
umlaut	⠠⠠⠠	(46,25)	¨

5.0.13 OPERATION SIGNS (continued)

Cartesian product	⠠⠠⠠	(46,246)	
global Cartesian product	⠠⠠⠠⠠	(456,46,246)	
global intersection	⠠⠠⠠⠠	(456,4,134)	
global union	⠠⠠⠠⠠	(456,4,136)	
join	⠠⠠⠠	(4,1236)	
meet	⠠⠠⠠	(4,1234)	
minus or plus	⠠⠠⠠	(456,36)	
plus or minus	⠠⠠⠠	(456,346)	
union	⠠⠠⠠	(4,136)	

5.0.14 PUNCTUATION (continued)

ellipsis, vertical	⠠⠠	(456,2)	
--------------------	----	---------	--

5.0.15 SPECIAL LETTERS

5.0.15.1 CROSSED

b	⠠⠨	(4,12)
d	⠠⠠⠨	(4,145)
h	⠠⠠⠠⠨	(4,125)
λ (lambda)	⠠⠠⠠⠠⠨	(46,5,123)

5.0.15.2 GLOBAL OPERATION SIGNS

global product	⠠⠠⠠⠠⠠⠠	(456,6,1234)	Π
global sum	⠠⠠⠠⠠⠠	(456,6,234)	Σ

5.0.15.3 HEBREW

aleph	⠠⠠⠠	(46,2)	\aleph
beth	⠠⠠⠠	(46,23)	\beth

5.0.15.4 MISCELLANEOUS

partial derivative	⠠⠠⠠	(4,1246)	∂
Weierstrass p	⠠⠠⠠⠠⠠	(46,5,1234)	

5.0.15.5 NUMBER SETS

complex	⠠⠠⠠⠠⠠	(456,456,14)
integers	⠠⠠⠠⠠	(456,456,24)
natural	⠠⠠⠠⠠	(456,456,1345)
rational	⠠⠠⠠⠠⠠	(456,456,1356)
real	⠠⠠⠠⠠	(456,456,1235)

5.0.15.6 OBSOLETE GREEK

koph or koppa	⠠⠠⠠⠠⠠	(46,5,12345)
sampi	⠠⠠⠠⠠	(46,5,156)

5.0.15.7 VARIANT GREEK

alpha	⠠⠠⠠⠠	(46,5,1)
beta	⠠⠠⠠⠠	(46,5,12)
theta	⠠⠠⠠⠠⠠	(46,5,1456)
sigma	⠠⠠⠠⠠⠠	(46,5,234)
phi	⠠⠠⠠⠠	(46,5,124)

5.0.16 TRIGONOMETRY

arcsecant	arcsec
arccosine	arccos
arccotangent	arccot
arccotangent	arcctn
arcsecant	arcsec
arcsine	arcsin
arctangent	arctan

cosecant	csc
cosine	cos
cotangent	cot
cotangent	ctn
cosine	cos
cotangent	cot
cotangent	ctn
hyperbolic cosecant	csch
hyperbolic cosine	cosh
hyperbolic cotangent	coth
hyperbolic cotangent	ctnh
hyperbolic secant	sech
hyperbolic sine	sinh
hyperbolic tangent	tanh
inverse cosecant	csc^{-1}
inverse cosine	cos^{-1}
inverse cotangent	cot^{-1}
inverse cotangent	ctn^{-1}
inverse hyperbolic cosecant	csch^{-1}
inverse hyperbolic cosine	cosh^{-1}
inverse hyperbolic cotangent	coth^{-1}
inverse hyperbolic cotangent	ctnh^{-1}
inverse hyperbolic secant	sech^{-1}
inverse hyperbolic sine	sinh^{-1}
inverse hyperbolic tangent	tanh^{-1}
inverse secant	sec^{-1}
inverse sine	sin^{-1}
inverse tangent	tan^{-1}
radian	rad
secant	sec
sine	sin
tangent	tan

5.1 Arrows (continued)

In Chapter 3 we showed and discussed only horizontal and vertical arrows. There are, however, oblique arrows which are found mostly in chemistry, and these have been listed in Section 5.0.1 of the Mathematics Symbol Set.

Arrows are also used as modifiers in vector notation. NUBS permits these arrows to be omitted if there is other evidence that the notation represents a vector. However, omission of such arrows is optional--not mandatory.

5.2 The ASCII Character Set

NUBS does not require a separate set of symbols for computer or calculator notation. The current Computer Braille Code was devised with the underlying assumption that every character in the basic ASCII Character Set should be represented by a one-cell braille character. Since there are 95 printable characters in the basic ASCII Character Set but only 63 (64 if the space is counted) distinct braille characters in our 6-dot braille system, this goal of one-to-one correspondence between ASCII characters and braille characters is, *prima facie*, impossible to achieve. NUBS maintains that a strict one-to-one correspondence is unnecessary and proceeds accordingly.

The basic ASCII Character Set, excluding lower-case and upper-case letters and excluding digits, is shown together with their corresponding tactile graphics in Section 5.0.2 of the Mathematics Symbol Set.

Here is an example of a computer program in C++ to show how NUBS handles such a program. It is a binary search program to decide whether a particular value, x , occurs in an array v which contains n elements arranged in ascending order and numbered from 0 to $n-1$. The function *binary* returns a number between 0 and $n-1$ if a match is found between x and an element of v . This number represents the position in v where the match was found. If no match is found, the function *binary* returns a value of -1.

EXAMPLE (of a C++ computer program)

```
binary(x, v, n) /*find the position in v that matches x*/
int x, v[ ], n;
{int low, high, mid;
  low = 0;
  high = n-1;
  while (low <= high)
  {mid = (low+high)/2;
   if (x < v[mid])
     high = mid-1;
   else if (x > v[mid])
     low = mid+1;
   else /*found match*/
     return(mid);
  }return(-1);
}
```

With this implementation, several braille characters remain unassigned to ASCII characters. These are:

```
⠆ ⠇ ⠈ ⠉ ⠊ ⠋ ⠌ ⠍ ⠎
⠏ ⠑ ⠒ ⠓ ⠔ ⠕ ⠖ ⠗ ⠘
```

5.2.1 Auxiliary Notation

To assert that a single letter is upper case, use ⠠ (6) before that letter. This braille character is not assigned to any ASCII character, nor is it a prefix to any root. It is equivalent to holding down the shift key on a keyboard, entering the letter, and then releasing the shift key. This prefix may be used only before letters; it must not be used before any non-letter the way the shift key is used on a typewriter or on a computer keyboard.

The compound prefix ⠠⠠ (6,6) may be used to capitalize an entire word. If the word contains non-letters, it does not change the meaning of those non-letters. It resumes its effectiveness with the next letter after a non-letter symbol. Its effectiveness is terminated upon encountering the next delimiter. It is equivalent to setting Caps Lock to the On position on a keyboard and leaving it in that mode until the word has been completed, then reverting to the Caps Lock Off position.

The compound prefix ⠠⠠⠠ (6,6,56) may be used to introduce an upper-case passage. It does not change the meaning of non-letters within that passage. Its effect is cancelled by the ⠠ (6,3) indicator. It is equivalent to setting Caps Lock to the On position on a keyboard, then setting it to the Off position upon encountering the ⠠ (6,3) indicator.

Font switching in computer programs, either for emphasis or for any other purpose, is accomplished in the same way as it is accomplished in NUBS at large.

5.3 Calculus and Analysis

The signs that are closely related to the field of calculus and analysis are shown in Section 5.0.3 of the Mathematics Symbol Set.

The signs for vertical bar and double vertical bar belong to the category of Grouping Signs as well as to the category of Calculus and Analysis.

The root of the infinity sign is the digit 8, suggestive of the fact that it is an 8 on its side in print. It is sometimes referred to as the "lazy 8."

The integral sign frequently carries limits of integration as a subscript and a superscript. These sometimes have the appearance of underscript and overscript modifiers. However, the integral sign is too thin to support underscripts and overscripts. Therefore, despite appearances, these should be transcribed as subscripts and superscripts.

Contour integrals have various closed shapes superposed on them. When this occurs, the integral sign should be followed by the overstrike indicator ⠠⠠ (5,12346) which, in turn, should be followed by the superposed shape.

The letters in the three "limit" entries are printed in the host type as is the case for all function abbreviations. An alternative for the upper limit is the letters lim with an overscript bar. Similarly, the lower limit is represented by the letters lim with an underscript bar. All three limits may have other underscripts.

log and ln are function abbreviations printed in the host type. Most of the time, log without a subscript is understood to have 10 as its base. At other times, 10, e , or some other base may

be attached as a subscript to log. \ln without a subscript is always understood to have e as its base. A common transcriber error is to mistake \ln for ln . Note that both letters in \ln are in the host type. If ln were intended, n , being a variable, would be printed in the mathematical italic type. Noting this rather subtle distinction would avoid this error.

The principal use of the prime sign in calculus is to denote the order of the derivative. Thus, $f'(x)$ denotes the first derivative of $f(x)$, $f''(x)$ denotes the second derivative of $f(x)$, etc. After the third derivative, it is customary to use Roman numbers enclosed within parentheses as superscripts to denote the order of the derivative. Thus $f^{(iv)}(x)$ denotes the fourth derivative of $f(x)$.

All the function abbreviations in trigonometry are also prominent in calculus.

Nabla is an inverted upper-case delta. Therefore, in NUBS, the roots of delta and nabla are up-down symmetric as these symbols are in print. They both have the same prefix.

The partial derivative sign is sometimes referred to as "round d."

5.4 Comparison Signs (continued)

The basic comparison signs were presented in Chapter 3, Section 3.0.3. Other comparison signs, together with their tactile graphics, are presented in Section 5.0.4 of the Mathematics Symbol Set.

In the signs for "less than or equal" and "greater than or equal," the bottom line is sometimes horizontal instead of parallel to the line above it; sometimes it is replaced by a complete equals sign. NUBS does not distinguish between these variant notational forms of these signs; the differences have no notational significance.

The stroke in the negative comparison signs is sometimes oblique and sometimes vertical. NUBS does not recognize this difference since it has no notational significance.

Comparison signs are sometimes called relational operators.

5.5 Computer and Calculator

Most modern arithmetic and algebra textbooks and computer and calculator manuals present a sequence of keystrokes by displaying a row of keytop legends. Legends other than digits are printed on a picture of a keytop. Digits are often displayed without being superposed on the picture of a keytop.

NUBS provides a method for showing a sequence of keystrokes when such a sequence is present in print. A keystroke which is the picture of a keytop with its legend is introduced in NUBS by the braille character $\cdot\textcircled{\cdot}$ (246). In other notational contexts, this character is either the termination indicator or the end-cancellation indicator. When it is the first character of a keystroke, it cannot be interpreted as a termination indicator. Neither can it be interpreted as an end-cancellation indicator which is always placed either above or below the notation to be cancelled. Therefore, using this symbol to introduce a keystroke is unambiguous.

A *simple fraction* is one in which neither the numerator nor the denominator is (or contains) a fraction. A simple fraction is said to be of order 0. For purposes of NUBS only, the order of a fraction is not increased if the only fractions that it contains are at the subscript level, the superscript level, or both.

A *complex fraction* is one in which the numerator, the denominator, or both are (or contain) only simple fractions. A complex fraction is said to be of order 1. The term "complex" is also used in a generic sense to apply to any fraction of order higher than 0. That is, "complex" is used as the opposite of "simple" when referring to fractions.

A *hypercomplex fraction* is one in which the numerator, the denominator, or both are (or contain) a complex fraction. A hypercomplex fraction is said to be of order 2.

In theory, it is possible to have fractions of any order whatever. As a practical matter, we frequently encounter complex fractions, we sometimes encounter hypercomplex fractions, and we almost never encounter any fractions of higher order than 2. Continued fractions (discussed in Section 5.7.3) are an exception to this statement, since they frequently are of order greater than 2. For purposes of NUBS only, we do not assign any order of complexity to a continued fraction.

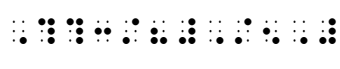
Since a fraction of higher order contains more than one fraction bar, it is necessary to identify the main fraction bar. In print, this is usually the fraction bar that is the longest in the entire construct, the one that is abreast of a reference label, a comparison sign, or some other sign of operation, or the one which is printed in type heavier than the others.

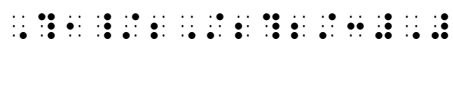
NUBS provides a *complexity indicator* to inform the reader of the complexity of the fraction with which he is dealing. This is another instance in which NUBS implements the *just-in-time principle*.

RULES (regarding the complexity of a fraction)

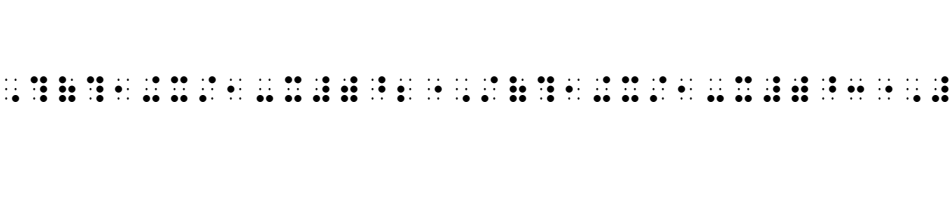
- a *These rules apply only when representing a fraction linearly.*
- b *The complexity indicator must not be used with simple fractions.*
- c *The complexity indicator must be used once before the begin-fraction indicator, the end-fraction indicator, and the fraction bar in the case of a complex (first order) fraction.*
- d *The complexity indicator must be used twice before the begin-fraction indicator, the end-fraction indicator, and the fraction bar in the case of a hypercomplex (second order) fraction.*
- e *In general, in the case of a fraction of order n , the complexity indicator must be used n times before the begin-fraction indicator, the end-fraction indicator, and the fraction bar of such a fraction. Fractions of order greater than 2 are almost never encountered.*

EXAMPLES (regarding complex fractions)

$$1 \quad \frac{\frac{3}{8}}{5}$$


$$2 \quad \frac{1/2}{2\frac{2}{3}}$$


$$3 \quad \frac{2/3}{3/2}$$


$$4 \quad \frac{\left(\frac{1+x}{1-x}\right)^2}{\left(\frac{1+x}{1-x}\right)^3}$$


Example 1 is a complex fraction whose numerator is a simple fraction.

Example 2 contains a mixed number in its denominator. The fractional part of this mixed number is a simple fraction, causing the entire construct to be treated as a complex fraction.

Example 3, despite appearances, is a simple fraction. The slashes are merely signs of operation and are treated just like a plus sign or a minus sign might be.

In Example 4, the presence of the parentheses does not obscure the fact that there are simple fractions both in the numerator and in the denominator, so that the entire construct is a complex fraction and is so presented.

5.7.2 Spatial Representation of Fractions

NUBS provides for the representation of fractions in a form which simulates their appearance in print. Fractions presented in this way are called *spatial fractions*, and the fraction bar in such fractions is called a *spatial fraction bar*.

There are several situations in which it is appropriate to represent fractions spatially:

- When fractions are first introduced to young children and for a short while thereafter;
- When the structure of a complex fraction is more clearly displayed spatially than linearly;
- In the representation of a continued fraction;
- When performing cancellation or other computation with fractions.

Although continued fractions are spatial, they are excluded from this discussion.

RULES (regarding spatial fractions)

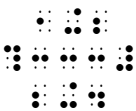
- a *A spatial fraction is a spatial arrangement; it must be separated from surrounding text or from other spatial arrangements by a blank line both above and below the spatial fraction.*
- b *A spatial fraction bar must be simulated by one or more braille characters composed of dots 25. This series of components must be long enough to cover the longer of the terms of the fraction.*
- c *A spatial fraction must be positioned so that its main spatial fraction bar is abreast of a label, a comparison sign, or a sign of operation, if these are present.*
- d *The begin-fraction and the end-fraction indicators at the ends of spatial fraction bars must be represented without complexity indicators; the spatial disposition of the fraction and the complexity indicators in its subsidiary linear fractions (if any) supply the requisite information to the braille reader.*
- e *It is permissible for a spatial fraction to have subsidiary fractions (of lower order) which are represented linearly.*
- f *When subsidiary fractions are represented linearly, they must be represented as self-contained local entities, without regard to the larger structure of which they are a part. In particular, complexity indicators must be used only as they apply to the linear portion of the notation.*
- g *The first character in each term of a fraction must be aligned with the first component of the spatial fraction bar with which it is associated.*
- h *A runover, when required, must be indented two cells beyond the expression of which it is a part.*
- i *If a term in a spatial fraction is a single digit, that digit must be preceded by the numeric indicator ⠠ (3456). If a term in a spatial fraction is an isolated lower-case letter from a to j (occupying a single cell), that letter must be preceded by the notational indicator ⠨ (56). In these cases, the numeric indicator and the notational indicator serve as dot locators.*

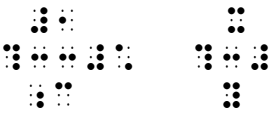
Note that the rule regarding the centering of the terms in a spatial fraction and their possible runovers has been changed from the 1972 Nemeth Code. There are also some other minor changes from the 1972 Nemeth Code.

EXCEPTION

Fraction indicators must not be used in a continued fraction.

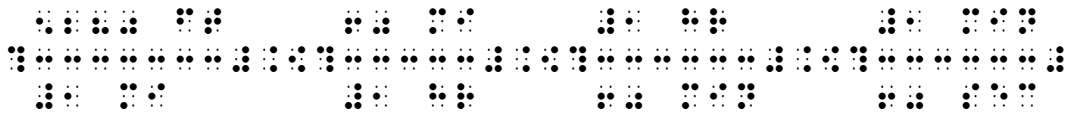
EXAMPLES (regarding spatial simple fractions)

1 $\frac{1+2}{2+4}$ 

2 $\frac{1}{c}, \frac{x}{y}$ 

3 $rate = \frac{distance}{time}$ 

4 $\frac{5280 ft}{1 mi} \times \frac{60 mi}{1 hr} \times \frac{1 hr}{60 min} \times \frac{1 min}{60 sec} = \frac{88 ft}{1 sec} = 88 ft/sec$





Example 1 is the kind that might be encountered in an elementary grade when fractions are first being introduced to young children.

Example 2 shows how isolated digits and letters are handled.

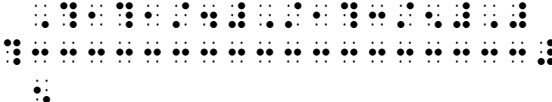
Example 3 consists of words. But since it is displayed, it is notational so that it contains no contractions.

Example 4 is a typical example of a technique called "dimensional analysis." When these fractions are displayed spatially rather than linearly, it is easy to see that each unit abbreviation in the denominator of one fraction "cancels" the same unit abbreviation in the numerator of the next fraction. The only abbreviations that remain are the "ft" in the numerator of the first fraction and the "sec" in the denominator of the last fraction. With this spatial display, it is also easy to see that 60 occurs once among the numerators and twice among the denominators. By cancelling one pair of 60's we are left with 5280 in the numerator and 60 in the denominator. Doing the arithmetic of dividing 5280 by 60 (either mentally, or on a piece of paper, or on a calculator), we obtain 88 as the numerical result.

Putting all of this together, we find that 60 miles per hour (the second fraction) is the same as 88 feet per second.

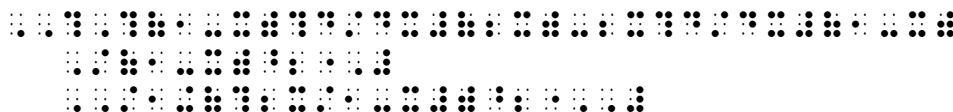
EXAMPLES (regarding spatial complex fractions)

1

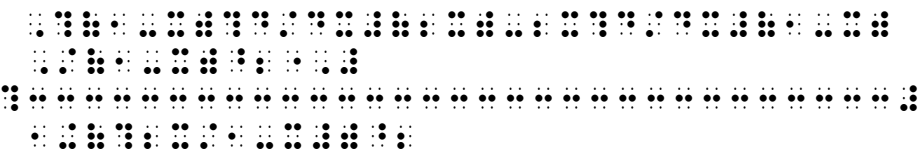
$$\frac{1 - \frac{1}{4}}{5 - \frac{3}{5}}$$


2

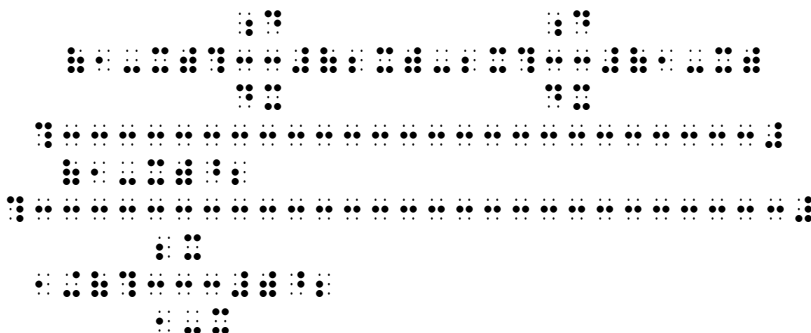
$$\frac{(1-x) \frac{d}{dx}(2x) - 2x \frac{d}{dx}(1-x)}{(1-x)^2} \div 1 + \left(\frac{2x}{1-x}\right)^2$$



3



4



Example 1 is the spatial representation of a hypercomplex fraction. The numerator is the quotient of two mixed numbers and so, in itself, is a complex fraction. This complex fraction is divided by 5, making the entire construct a hypercomplex fraction.

Examples 2, 3, and 4 are alternative ways of representing the same print information. In Example 2 the representation is entirely linear even though it is included in a group identified

as spatial complex fractions. Example 3 is partly linear and partly spatial. Example 4 is entirely spatial.

When requested to find

$$\frac{d}{dx} \tan^{-1} \frac{2x}{1-x}$$

this notation is the first step in working out the solution to finding this derivative.

5.7.3 Continued Fractions

A *continued fraction* is one in which each denominator except possibly the last one is the sum of a whole number and a fraction. See example below.

RULES (regarding continued fractions)

- a *A continued fraction must be represented spatially.*
- b *A continued fraction is a spatial arrangement and thus requires a blank line above and below it.*
- c *A continued fraction must be positioned so that its first fraction bar is abreast of a label, a comparison sign, or other operation sign, if any.*
- d *Fraction indicators must not be used in the representation of a continued fraction.*
- e *Each fraction bar must be long enough to cover all of the notation below it which is part of the continued fraction.*

EXAMPLE (of a continued fraction)

$$\sqrt{2} = 1 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \dots}}}}$$

$$\begin{aligned}
x_5 &= 2.414285714286 \\
x_6 &= 2.414201183432 \\
x_7 &= 2.414215686275 \\
x_8 &= 2.414213197970 \\
x_9 &= 2.414213624895 \\
x_{10} &= 2.414213551646 \\
x_{11} &= 2.414213564214 \\
x_{12} &= 2.414213562057 \\
x_{13} &= 2.414213562427 \\
x_{14} &= 2.414213562364 \\
x_{15} &= 2.414213562375 \\
x_{16} &= 2.414213562373 \\
x_{17} &= 2.414213562373
\end{aligned}$$

Remember that these are successive approximations to $1+\sqrt{2}$. Note that the approximations alternate between being smaller and larger than the true value of $1+\sqrt{2}$. We stop when two successive approximations are equal; at that point we have reached the precision limit of our calculator. In this case, we have the approximation of $1+\sqrt{2}$ correct to 12 digits after the decimal point. Subtracting 1 from the value of x_{17} , we have the value of $\sqrt{2}$ correct to 12 digits after the decimal point, namely:

$$\sqrt{2} = 1.414213562373$$

This is by no means the fastest way of computing $\sqrt{2}$. It is, however, an interesting method. Furthermore, the continued fraction for $\sqrt{2}$ was given by revelation; no attempt was made to show how we arrived at this continued fraction. If you had to compute $\sqrt{3}$ by using a continued fraction, you would first have to find the continued fraction that represents $\sqrt{3}$. This book is not the place to teach you how to do that.

5.7.4 Indexed Radicals

A radical is said to be *indexed* if it carries a superscript. If the superscript is 3, we are dealing with the cube root; if the superscript is n , we are dealing with the n^{th} root, etc. When a radical sign carries no index, the index is implicitly understood to be 2, so in that case, we are dealing with the square root.

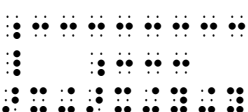
In NUBS, the index of a radical is attached to the radical sign in the form of a superscript. Note that this differs from the method of indexing a radical as provided in the 1972 Nemeth Code.

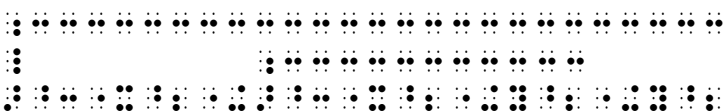
RULE (regarding indexed radicals)

If a radical is indexed, the index must be attached to the radical sign in the form of a superscript.

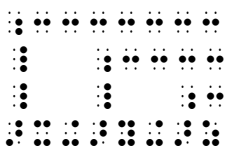
- e Each vinculum is simulated by a horizontal row of dots in which each component consists of dots 25. The vinculum must occupy the same braille line as the topmost component of its ascender. Its first component must occupy the cell after the topmost component of its ascender, and the vinculum must be long enough to cover the radicand with which it is associated.
- f The order of the radical is not indicated in a spatial representation, since the extent and position of each ascender and vinculum provides this information.

EXAMPLES (showing the spatial representations of the four examples of Section 5.7.5)

1a $\sqrt{x + \sqrt{x + y} + y}$ 

2a $\sqrt[3]{x^2 + \sqrt[3]{x^2 + y^2} + y^2}$ 

3a $\sqrt{\sqrt[3]{x}} = \sqrt[3]{\sqrt{x}}$ 

4a $\sqrt{x + \sqrt{y + \sqrt{z}}}$ 

5.7.7 Fractions and Radicals Together in a Spatial Representation

It is possible to have spatial arrangements in which radicals are contained within fractions or in which fractions are contained within radicals. The method is straightforward; nothing more is required than to apply the rules for the representation of spatial fractions and spatial radicals. We present one example as a model.

Upper half-brackets are used in mathematics for the "ceiling" function, and lower half-brackets are used for the "floor" function. These half-brackets are also used for other miscellaneous purposes. In the mathematics of finance, the upper right half-bracket is used as a subscript with an expression inside the bracket and an expression outside and to the right of the bracket.

EXAMPLE (from the mathematics of finance)

19 $\lceil A_n \rceil_i$

Extended parentheses are used to enclose a binomial coefficient. In print, the components of a binomial coefficient are written one above the other. In NUBS, a binomial coefficient is represented by an extended set of parentheses in which the upper component is followed by the lower component, separated by a space.

EXAMPLE (of binomial coefficient)

20 $\binom{n!}{(n-k)!}$

In print, extended grouping signs are used to span a determinant, a matrix, or a system of equations. In NUBS, the extended grouping signs should be placed on every line to the left, to the right, or on both sides of the notation which it spans, following print. Extended grouping signs should not be used to span a vertically-stacked fraction or to span a tall integral sign which spans a vertically-stacked fraction, or a stacked binomial coefficient. If the system with its extended grouping signs is associated with an identifier, a comparison sign, or an operation sign, it should be so positioned that its top line is abreast of these.

RULE (for the placement of a system spanned by extended grouping signs)

A system spanned by extended grouping signs should be placed so that its top line is abreast of a label, a comparison sign, or an operation sign, if any. If the system is punctuated, the punctuation mark should be placed alongside the top line.

EXAMPLES (of extended grouping signs)

21 $\begin{cases} x + y = 2 \\ x - y = 0 \end{cases}$

(a system of two equations spanned on the left by an extended brace)

22 $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

(The evaluation of a 2 x 2 determinant is enclosed by vertical bars. The right side of the equation in print is centered on the extended vertical bar; in NUBS it is abreast of the top line of the extended vertical bar.)

$$23 \quad y = \begin{cases} x, & \text{if } x \leq 0 \\ 0, & \text{if } x > 0 \end{cases}$$

(a piecewise-defined equation whose components are spanned on the left by an extended bracket)

$$24 \quad x = \begin{bmatrix} \cos a & \sin a & 0 \\ -\sin a & \cos a & 0 \\ 0 & 0 & 1 \end{bmatrix} .$$

(A 3 x 3 matrix is spanned on both sides by extended brackets. Isolated digits require the numeric indicator. In print, the left side of the equation is centered on the extended left bracket. The period on the right is centered on the extended right bracket. In NUBS, both of these are abrest of the top line of the matrix.)

Sometimes a system of equations or a piecewise-defined equation has a comment, narrative or notational, centered so as to apply to the system, but without any extended grouping sign at all. When this happens, NUBS requires an extended transcriber enclosure to span the system so as to associate the comment with the system. If the comment is to the left of the system, the left extended transcriber enclosure is required; if at the right, the right extended transcriber enclosure is required.

EXAMPLE (showing the use of an extended transcriber enclosure)

$$25 \quad a = \frac{x+y}{x-y}$$

$$\begin{aligned} -1 < x < 1, \\ -1 < y < 1 \end{aligned}$$

$$b = \frac{x-y}{x+y}$$

(In print, the conditions for x and y are on two lines centered between two equations with no extended grouping sign.)

5.11 Logic and Set Theory

Logic and set theory are closely related branches of mathematics and thus share many symbols. Most of those symbols can be classified either as operation signs or as comparison signs, and have been included in their respective categories in the Mathematics Symbol Set together with their tactile graphics. They are governed by the same rules that we have already encountered regarding other operation signs and comparison signs.

Note that the symbols for "meet" and "join" are top-to-bottom symmetric in print; they are likewise top-to-bottom symmetric in NUBS. The same observation holds for the signs for "union" and "intersection."

Note that the prefix \neg (46) denotes negation. It is placed either before the first character of the principal sign to achieve negation, or it replaces that first character.

The comparison signs in the Logic and Set Theory list have been duplicated in the Signs of Comparison list, and the operation signs in the Logic and Set Theory list have been duplicated in the Operation Signs list. Four entries in the Logic and Set Theory list are neither comparison signs nor operation signs.

5.12 Special Letters (continued)

The special letters recognized by NUBS have been assigned to seven categories. These are all listed in the Mathematics Symbol Set together with their graphic representation.

5.12.1 Crossed

The crossed b is used in computer displays to denote a blank when it is important to count the actual number of blanks.

The crossed h is used to represent Planck's Constant in the field of quantum mechanics.

5.12.2 Global Operation Signs

The large upper-case boldface sigma is used to represent the sum of a large number of terms. Similarly, the large upper-case boldface pi is used to represent the product of a large number of factors. Other global operation signs that are not letters are entries in the Logic and Set Theory list.

5.12.3 Hebrew

Two Hebrew letters, aleph and beth, are used in the branch of mathematics that deals with transfinite numbers. The 1972 Nemeth Code lists letters of the entire Hebrew alphabet, both in their printed and script forms. However, experience has proved this to be unnecessary.

5.12.4 Miscellaneous

The two letters, partial derivative and Weierstrass p , are used in the branch of mathematics that deals with calculus and analysis.

5.12.5 Number Sets

Mathematics employs five letters, each in the same special font, to denote the principal types of numbers: the natural numbers, the integers, the rational numbers, the real numbers, and the complex numbers.

5.12.6 Obsolete Greek

Obsolete Greek letters are used in discussions concerning the ancient Greek numeration system.

5.12.7 Variant Greek

Sometimes two forms of the same Greek letter are used in an expository passage and in the accompanying notational text. When this is the case, the variant form must be distinguished from the standard form, so that NUBS provides the means for doing so.

At other times, a variant form of a Greek letter is used without mention of the alternative standard form. When this is the case, NUBS permits the variant form to be replaced by the standard form of that letter without fear of ambiguity.

5.13 Subscripts and Superscripts (continued), and Modifiers

It is characteristic of mathematical notation as well as of notation elsewhere to place some symbols at an elevated or a depressed position relative to the principal line of notation. Symbols that are elevated are called *superscripts*; symbols that are depressed are called *subscripts*. The principal line of notation is called the *base line*. Notation, either at an elevated level or at a depressed level, is called an *index* (plural, *indices*).

Subscripts and superscripts have been mentioned in an informal way in Chapter 3 as well as in several earlier sections of this chapter. It is time to give them a coherent analysis.

A superscript is introduced by the indicator $\overset{\cdot}{\cdot}$ (45). It remains in effect until it is cancelled by another level indicator or by a space. Similarly, a subscript is introduced by the indicator $\underset{\cdot}{\cdot}$ (56). It remains in effect until it is cancelled by another level indicator or by a space. The indicator $\overset{\cdot}{\cdot}$ (5) is used to return to the base line. Notice how suggestive these indicators are of their functions.

EXAMPLES (of simple subscripts and superscripts)

1 x^2 $\overset{\cdot}{\cdot} \overset{\cdot}{\cdot} \overset{\cdot}{\cdot} \overset{\cdot}{\cdot}$ (x square)

Trigonometry is characterized by the generous use of function abbreviations. It is customary for these function abbreviations to be printed in the host type. The function abbreviation for cotangent has two commonly used forms, namely, \cot and ctn . There are other function abbreviations which are not specific to trigonometry, and these have been listed separately in Section 5.0.7 of the Mathematics Symbol Set. Other than these function abbreviations, the notation in trigonometry is the same as that used in other branches of mathematics.

Some now obsolete trigonometric function abbreviations are included in the list of Function Abbreviations.

This index cover the entire NUBS document, Chapters 0 through 8.

Each entry in the index includes the section number and the print page number. In the braille version, this is followed by an additional notation to provide the braille volume and braille page number. For example a reference of: ANGLE BRACKETS 3.2.3 49 indicates that angle brackets are addressed in Section 3.2.3 on print page 49. Since the indexed item may not occur at the beginning of the referenced section, the print page listed may not include the start of that section.

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